Introduction to Neutron Physics)

Developed for the Members of the

Global Sustainability Alliance

By: Joseph D. Fournier B.Sc.E.E., M.Sc.E.E.

Preamble

The neutron is essential to the production of nuclear power. In the decade after the neutron was discovered by James Chadwick in 1932, neutrons were used to induce many different types of nuclear transmutations.

With the discovery of nuclear fission in 1938, it was quickly realized that, if a fission event produced neutrons, each of these neutrons might cause further fission events, in a cascade known as a nuclear chain reaction.

These events and findings led to the first self-sustaining nuclear reactor (Chicago Pile-1, 1942).

Outline

- Discovery of the Neutron
- Nuclear Properties
- The Deuteron
- Nuclear Forces
- Nuclear Stability
- The Liquid Drop Model
- Radioactive Decay
- Alpha, Beta, and Gamma Decay
- Radioactive Nuclides

1: Discovery of the Neutron

- Rutherford proposed the atomic structure with the massive nucleus in 1911.
- Scientists knew which particles compose the nucleus in 1932.
- Reasons why electrons cannot exist within the nucleus:

1) **Nuclear size**
   The uncertainty principle puts a lower limit on its kinetic energy that is much larger that any kinetic energy observed for an electron emitted from nuclei.
2) **Nuclear spin**
   If a deuteron consists of protons and electrons, the deuteron must contain 2 protons and 1 electron. A nucleus composed of 3 fermions must result in a half-integral spin. But it has been measured to be 1.

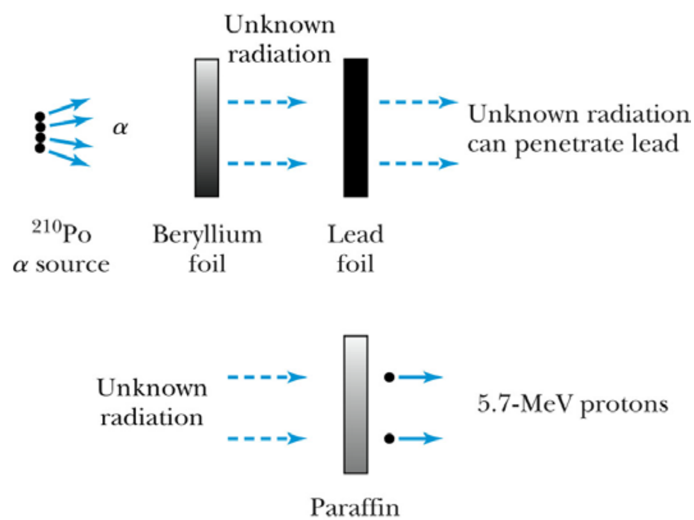
1: Discovery of the Neutron

3) Nuclear magnetic moment:

The magnetic moment of an electron is over 1000 times larger than that of a proton.

The measured nuclear magnetic moments are on the same order of magnitude as the proton's, so an electron is not a part of the nucleus.

- In 1930 the German physicists Bothe and Becker used a radioactive polonium source that emitted ɑ particles. When these ɑ particles bombarded beryllium, the radiation penetrated several centimeters of lead.



1: Discovery of the Neutron

- The electromagnetic radiation (photons) are called **gamma rays** which have energies on the order of MeV.
- Curie and Joliot performed several measurements to study penetrating high-energy gamma rays.
- In 1932 Chadwick proposed that the new radiation produced by α + Be consisted of neutrons. His experimental data estimated the neutron's mass as somewhere between 1.005 u and 1.008 u, not far from the modern value of 1.0087 u.

2: Nuclear Properties

- The nuclear charge is $+e$ times the number (Z) of protons.
- Hydrogen's **isotopes**:
  - **Deuterium**: Heavy hydrogen. Has a neutron as well as a proton in its nucleus.
  - **Tritium**: Has two neutrons and one proton.

- The nuclei of the deuterium and tritium atoms are called *deuterons* and *tritons*.
- Atoms with the same Z, but different mass number A, are called **isotopes**.

2: Nuclear Properties

- The symbol of an atomic nucleus is $^{A}_{Z}X_{N}$.

  where Z = atomic number (number of protons)

  N = neutron number (number of neutrons)

  A = mass number ($Z + N$)

  X = chemical element symbol

- Each nuclear species with a given Z and A is called a **nuclide**.
- Z characterizes a chemical element.
- The dependence of the chemical properties on N is negligible.
- Nuclides with the same neutron number are called *isotones* and the same value of A are called *isobars*.

2: Nuclear Properties

Table 12.1 Some Nucleon and Electron Properties

Particle	Symbol	Rest Energy (MeV)	Charge	Mass (u)	Spin	Magnetic Moment
Proton	p	938.272	$+e$	1.0072765	1/2	$2.79\ \mu_N$
Neutron	n	939.566	0	1.0086649	1/2	$-1.91\ \mu_N$
Electron	e	0.51100	$-e$	5.4858×10^{-4}	1/2	$-1.00116\ \mu_B$

- Atomic masses are denoted by the symbol u.
- $1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg} = 931.49\ \text{MeV}/c^2$
- Both neutrons and protons, collectively called **nucleons**, are constructed of other particles called *quarks*.

2: Sizes and Shapes of Nuclei

- Rutherford concluded that the range of the nuclear force must be less than about 10^{-14} m.
- Assume that nuclei are spheres of radius R.
- Particles (electrons, protons, neutrons, and alphas) scatter when projected close to the nucleus.
- It is not obvious whether the maximum interaction distance refers to the nuclear size (*matter radius*), or whether the nuclear force extends beyond the nuclear matter (*force radius*).
- The nuclear force is often called the **strong** force.
- Nuclear force radius ≈ mass radius ≈ charge radius

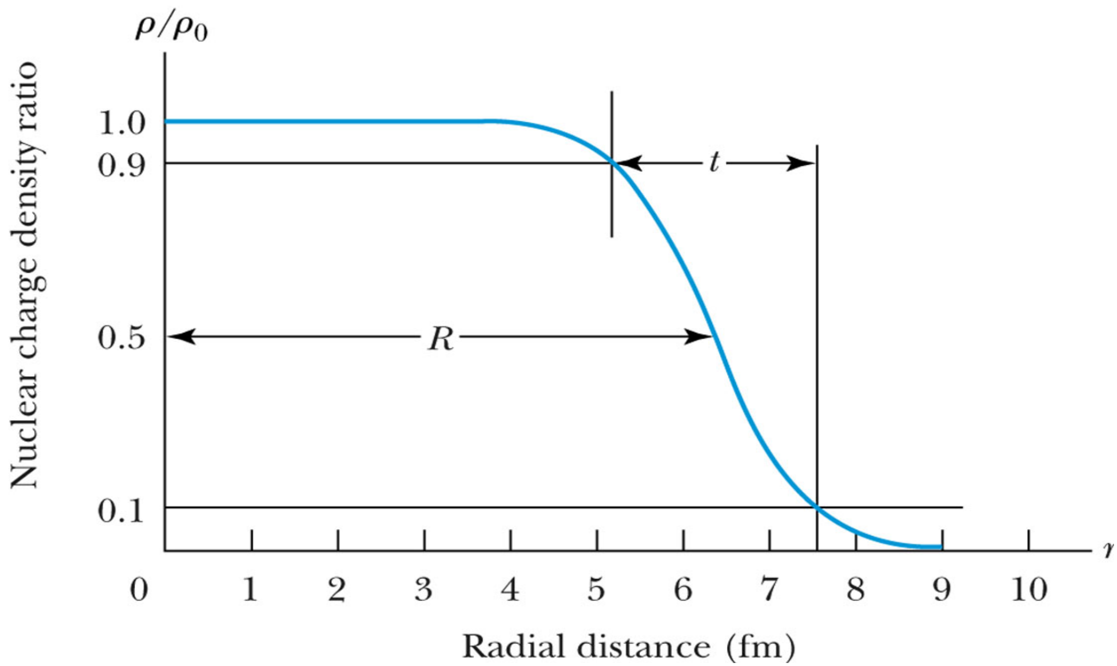
2: Sizes and Shapes of Nuclei

- The nuclear radius may be approximated to be $R = r_0 A^{1/3}$, where $r_0 \approx 1.2 \times 10^{-15}$ m.
- We use the **femtometer** with 1 fm = 10^{-15} m, or the fermi.
- The lightest nuclei by the Fermi distribution for the nuclear charge density $\rho(r)$ is

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

2: Sizes and Shapes of Nuclei

The shape of the Fermi distribution



- If we approximate the nuclear shape as a sphere,

$$V = \frac{4}{3}\pi r_0^{\,3} A$$

- The nuclear mass density is 2.3×10^{17} kg / m^3.

Intrinsic Magnetic Moment

- The proton's intrinsic magnetic moment points in the same direction as its intrinsic spin angular momentum.
- Nuclear magnetic moments are measured in units of the nuclear magneton μ_N.

$$\mu_N = \frac{e\hbar}{2m_p}$$

- The divisor in calculating μ_N is the proton mass m_p, which makes the nuclear magneton some 1800 times smaller than the Bohr magneton.
- The proton magnetic moment is $\mu_p = 2.79\mu_N$.
- The magnetic moment of the electron is $\mu_e = -1.00116\mu_B$.
- The neutron magnetic moment is $\mu_n = -1.91\mu_N$.
- The *nonzero* neutron magnetic moment implies that the neutron has negative and positive internal charge components at different radii.

→ Complex internal *charge distribution*.

3: The Deuteron

- The determination of how the neutron and proton are bound together in a deuteron.
- The deuteron mass = 2.013553 u.
- The mass of a deuteron atom = 2.014102 u.
- The difference = 0.000549 u. ⟶ the mass of an electron.
- The deuteron nucleus is bound by a mass-energy B_d.
- The mass of a deuteron is

$$m_d = m_p + m_n - B_d / c^2$$

- Add an electron mass to each side of Eq. (12.6)

$$m_d + m_e = m_p + m_n + m_e - B_d / c^2$$

3: The Deuteron

- $m_d + m_e$ is the atomic deuterium mass $M(^2\text{H})$ and $m_p + m_e$ is the atomic hydrogen mass. Thus Eq.(12.7) becomes

$$M(^2\text{H}) = m_n + M(^1\text{H}) - B_d / c^2$$

- Because the electron masses cancel in almost all nuclear-mass difference calculations, we use atomic masses rather than nuclear masses.

$$m_n = 1.008665 \text{ u} \qquad \text{Neutron mass}$$
$$M(^1\text{H}) = 1.007825 \text{ u} \qquad \text{Atomic hydrogen mass}$$
$$M(^2\text{H}) = 2.014102 \text{ u} \qquad \text{Atomic deuterium mass}$$
$$B_d / c = m_n + M(^1\text{H}) - M(^2\text{H}) = 0.002388 \text{ u}$$

- Convert this to energy using u = 931.5 MeV / c^2.

$$B_d = 0.002388\ c^2 \cdot \text{u}\left(\frac{931.5 \text{ MeV}}{c^2 \cdot \text{u}}\right) = 2.224 \text{ MeV}$$

- Even for heavier nuclei we neglect the electron binding energies (13.6 eV) because the nuclear binding energy (2.2 MeV) is almost one million times greater.

3: The Deuteron

- The binding energy of any nucleus $^{A}_{Z}X$ = the energy required to separate the nucleus into free neutrons and protons.

$$B\left({}^{A}_{Z}X\right) = \left[Nm_n + ZM\left({}^{1}\mathrm{H}\right) - M\left({}^{A}_{Z}X\right)\right]c^2$$

Experimental Determination of Nuclear Binding Energies

- Check the 2.22-MeV binding energy by using a nuclear reaction. We scatter gamma rays from deuteron gas and look for the breakup of a deuteron into a neutron and a proton:

$$\gamma + d \rightarrow n + p$$

- This nuclear reaction is called *photodisintegration* or a *photonuclear reaction*.
- The mass-energy relation is

$$hf + M({}^{2}\mathrm{H})c^2 = m_n c^2 + M({}^{1}\mathrm{H})c^2 + K_n + K_p$$

- where *hf* is the incident photon energy.

  K_n and K_p are the neutron and proton kinetic energies.

3: The Deuteron

- The minimum energy required for the photodisintegration:
- Momentum must be conserved in the reaction (K_n, $K_p \neq 0$).

$$hf_{\text{min}} = B_d\left[1 + \frac{B_d}{2M(^2\text{H})c^2}\right]$$

- Experiment shows that a photon of energy less than 2.22 MeV cannot dissociate a deuteron.

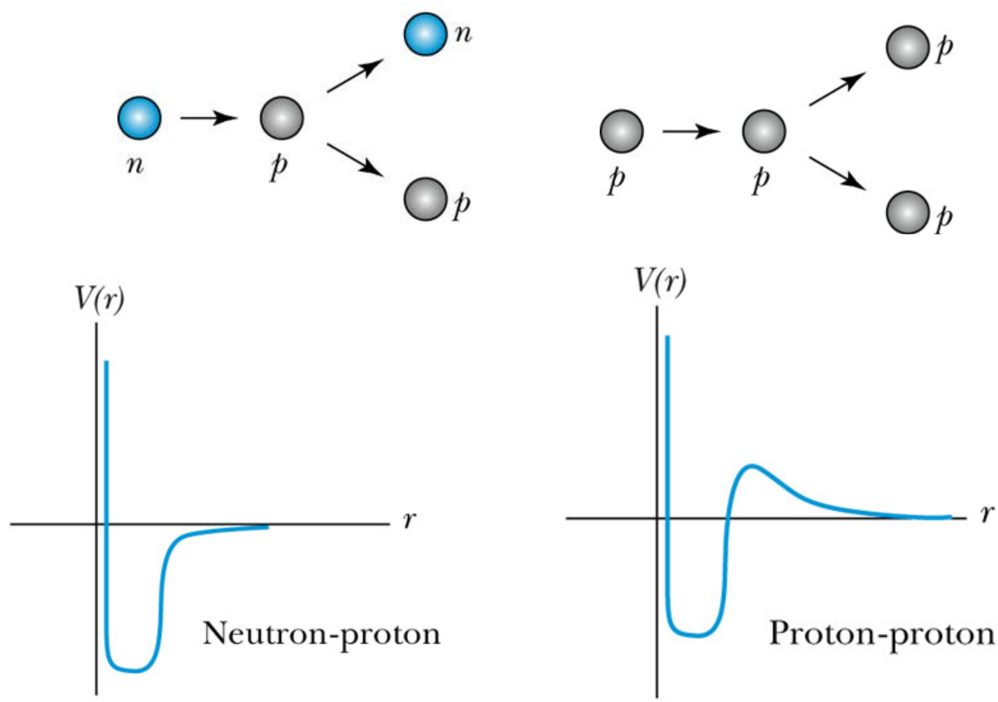
$$2.79\mu_{\text{N}} - 1.91\mu_{\text{N}} = 0.88\mu_{\text{N}}$$

Deuteron Spin and Magnetic Moment

- Deuteron's nuclear spin quantum number is 1. This indicates the neutron and proton spins are aligned parallel to each other.
- The nuclear magnetic moment of a deuteron is $0.86\mu_{\text{N}}$ ≈ the sum of the free proton and neutron $2.79\mu_{\text{N}} - 1.91\mu_{\text{N}} = 0.88\mu_{\text{N}}$.

4: Nuclear Forces

- The angular distribution of neutron classically scattered by protons.
- Neutron + proton (*np*) and proton + proton (*pp*) elastic.



The nuclear potential

4: Nuclear Forces

- The internucleon potential has a “hard core” that prevents the nucleons from approaching each other closer than about 0.4 fm.
- The proton has charge radius up to 1 fm.
- Two nucleons within about 2 fm of each other feel an attractive force.
- The nuclear force (*short range*): = **stable**.
- It falls to zero so abruptly with interparticle separation.
- The interior nucleons are completely surrounded by other nucleons with which they interact.
- The only difference between the *np* and *pp* potentials is the Coulomb potential shown for $r \geq 3$ fm for the *pp* force.

4: Nuclear Forces

- The nuclear force is known to be spin dependent.
- The neutron and proton spins are aligned for the bound state of the deuteron, but there is no bound state with the spins antialigned.
- The *nn* system is more difficult to study because free neutrons are not stable from analyses of experiments.
- The nuclear potential between two nucleons seems independent of their charge (*charge independence of nuclear forces*).
- The term *nucleon* refers to either neutrons or protons because the neutron and proton can be considered different charge states of the same particle.

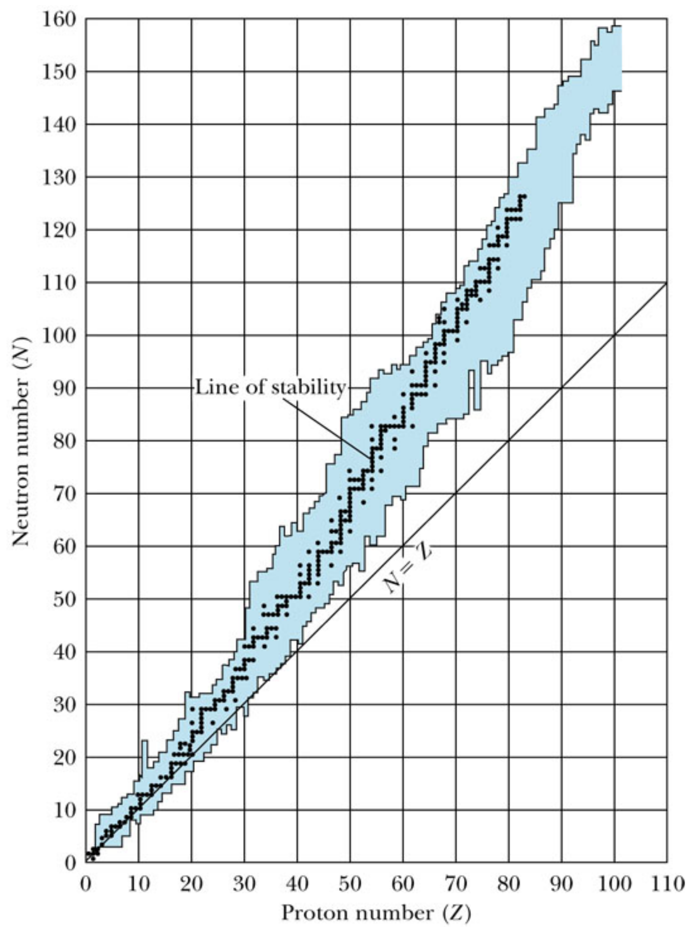
5: Nuclear Stability

- The binding energy of a nucleus against dissociation into any other possible combination of nucleons. Ex. nuclei *R* and *S*.

$$B = \left[M(R) + M(S) - M\left({}^{A}_{Z}X \right) \right] c^2$$

- Proton (or neutron) *separation energy*:
  - The energy required to remove one proton (or neutron) from a nuclide.

- All stable and unstable nuclei that are long-lived enough to be observed.



© 2006 Brooks/Cole - Thomson

5: Nuclear Stability

- The line representing the stable nuclides is the **line of stability**.
- It appears that for $A \leq 40$, nature prefers the number of protons and neutrons in the nucleus to be about the same $Z \approx N$.

  However, for $A \geq 40$, there is a decided preference for $N > Z$ because the nuclear force is independent of whether the particles are *nn*, *np*, or *pp*.

- As the number of protons increases, the Coulomb force between all the protons becomes stronger until it eventually affects the binding significantly.

- The work required to bring the charge inside the sphere from infinity is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}$$

5: Nuclear Stability

- For a single proton,

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}$$

- The total Coulomb repulsion energy in a nucleus is

$$\Delta E_{\text{Coul}} = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

- For heavy nuclei, the nucleus will have a preference for fewer protons than neutrons because of the large Coulomb repulsion energy.

- Most stable nuclides have both even *Z* and even *N* (even-even nuclides).
- Only four stable nuclides have odd *Z* and odd *N* (odd-odd nuclides).

$^{2}_{1}\text{H}$, $^{6}_{3}\text{Li}$, $^{10}_{5}\text{B}$, and $^{14}_{7}\text{N}$.

The Liquid Drop Model

- Treats the nucleus as a collection of interacting particles in a liquid drop.
- The total binding energy, the semi-empirical mass formula is

$$B\left({}_{Z}^{A}X\right) = a_V A - a_A A^{2/3} - \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 r} - a_S \frac{(N-Z)^2}{A} + \delta$$

- The volume term (a_V) indicates that the binding energy is approximately the sum of all the interactions between the nucleons.
- The second term is called the *surface effect* because the nucleons on the nuclear surface are not completely surrounded by other nucleons.
- The third term is the Coulomb energy.

The Liquid Drop Model

- The fourth term is due to the symmetry energy. In the absence of Coulomb forces, the nucleus prefers to have $N \approx Z$ and has a quantum-mechanical origin, depending on the exclusion principle.
- The last term is due to the pairing energy and reflects the fact that the nucleus is more stable for even-even nuclides. Use values given by Fermi to determine this term.

$$a_V = 14 \text{ MeV} \qquad \text{Volume}$$

$$a_A = 13 \text{ MeV} \qquad \text{Surface}$$

$$a_S = 19 \text{ MeV} \qquad \text{Symmetry}$$

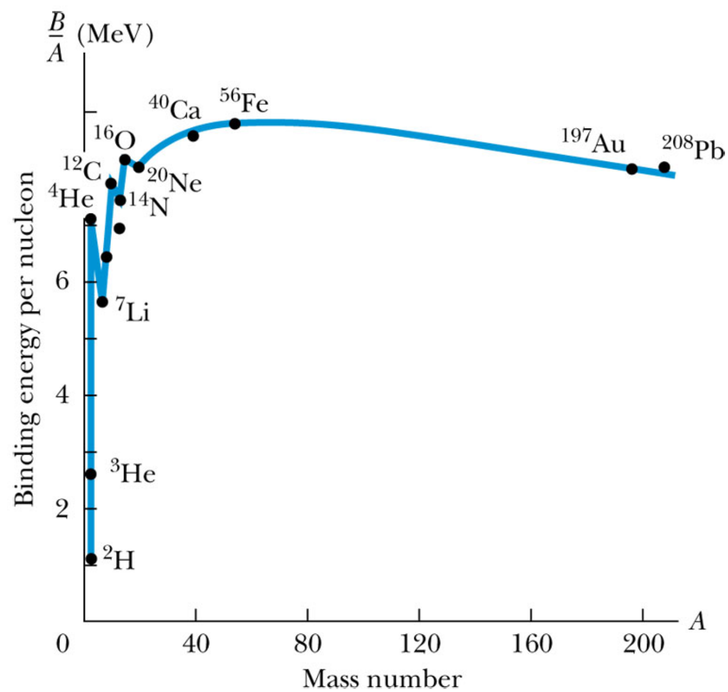
$$\text{Pairing } \delta = \begin{cases} +\Delta & \text{for even-even nuclei} \\ 0 & \text{for odd-}A\text{ (even-odd, odd-even) nuclei} \\ -\Delta & \text{for odd-odd nuclei} \end{cases}$$

where Δ = 33 MeV·$A^{-3/4}$.

- No nuclide heavier than $^{238}_{92}\text{U}$ has been found in nature. If they ever existed, they must have decayed so quickly that quantities sufficient to measure no longer exist.

Binding Energy Per Nucleon

- Use this to compare the relative stability of different nuclides.
- It peaks near A = 56.
- The curve increases rapidly, demonstrating the saturation effect of nuclear force.
- Sharp peaks for the even-even nuclides ^4He, ^{12}C, and ^{16}O $=$ tight bound.



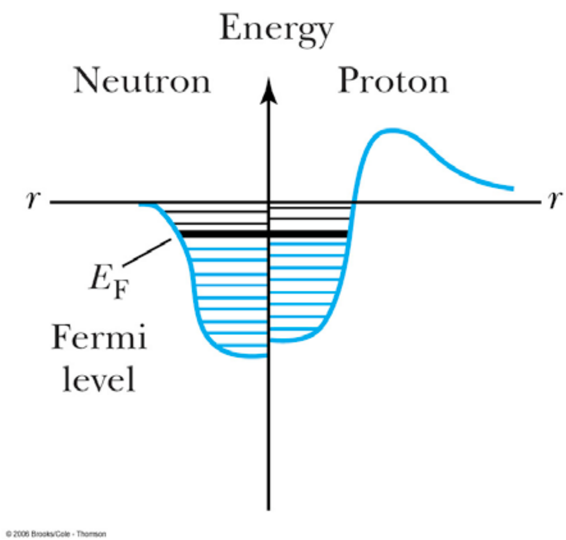
Nuclear Models

- Current research focuses on the constituent quarks and physicists have relied on a multitude of models to explain nuclear force behavior.

1) **Independent-particle models**:
   The nucleons move nearly independently in a common nuclear potential. The shell model has been the most successful of these.
2) **Strong-interaction models**:
   The nucleons are strongly coupled together. The liquid drop model has been successful in explaining nuclear masses as well as nuclear fission.

Nuclear Models

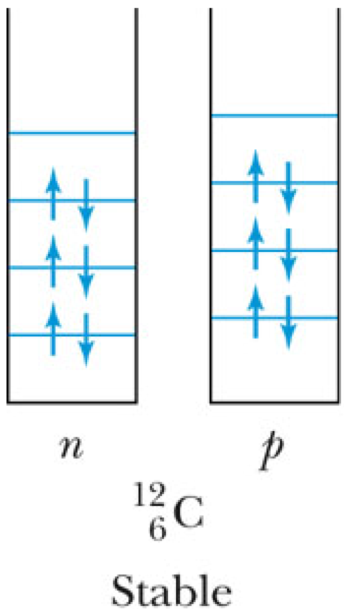
The nuclear potential felt by the neutron and the proton



- The difference of the shape between the proton and the neutron are due to the Coulomb interaction on the proton.
- Nuclei have a Fermi energy level which is the highest energy level filled in the nucleus.
- In the ground state of a nucleus, all the energy levels below the Fermi level are filled.

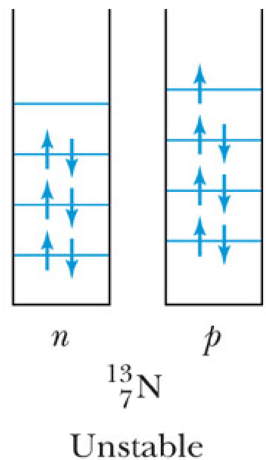
Nuclear Models

- Energy-level diagrams for ^{12}C .



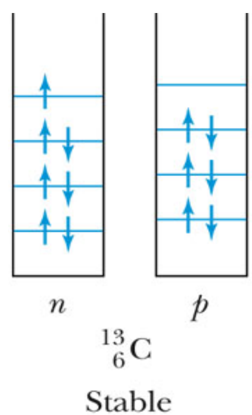
- Both are stable because they are even-even.

Case 1: If we add one proton to ^{12}C to make $^{13}_{7}N$



unstable

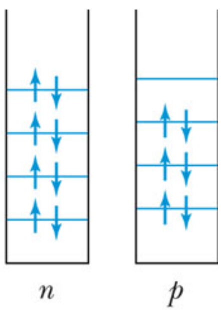
Case 2: If we add one neutron to ^{12}C to make ^{13}C:



stable

Nuclear Models

- Even when we add another neutron to produce ^{14}C, we find it is barely unstable.

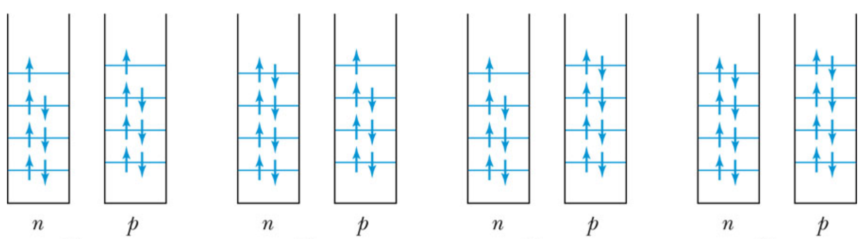


$^{14}_{6}$C
Unstable

- In this mass region, nature prefers the number of neutrons and protons to be N ≈ Z, but it doesn't want N ≈ Z.

This helps explain why ^{13}C is stable, but not ^{13}N.

- Indicating neutron energy levels to be lower in energy than the corresponding proton ones.



$^{14}_{7}$N Stable | $^{15}_{7}$N Stable | $^{15}_{8}$O Unstable | $^{16}_{8}$O Stable

6: Radioactive Decay

- Marie Curie and her husband Pierre discovered polonium and radium in 1898.
  - The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.
  - Other modes of decay include emission of α particles, β particles, protons, neutrons, and fission.

- The disintegrations or decays per unit time (**activity**).

$$\text{Activity} = -\frac{dN}{dt} = R$$

where dN / dt is negative because total number N decreases with time.

6: Radioactive Decay

- SI unit of activity is the becquerel: 1 Bq = 1 decay / s.
- Recent use is the Curie (Ci) 3.7×10^{10} decays / s.

- If N(t) is the number of radioactive nuclei in a sample at time *t*, and λ (**decay constant**) is the probability per unit time that any given nucleus will decay:

$$R = \lambda N(t)$$

$$dN(t) = -R\,dt = -\lambda N(t)\,dt$$

$$\int \frac{dN}{N} = -\int \lambda\,dt$$

$$\ln N = -\lambda t + \text{constant}$$

$$N(t) = e^{-\lambda t + \text{constant}}$$

- If we let $N(t = 0) \equiv N_0$

$$N(t) = N_0 e^{-\lambda t}$$ ----- **radioactive decay law**

ASCENTRUST LLC

6: Radioactive Decay

- The activity R is

$$R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where R_0 is the initial activity at $t = 0$.

- It is common to refer to the half-life $t_{1/2}$ or the mean lifetime τ rather than its decay constant.

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

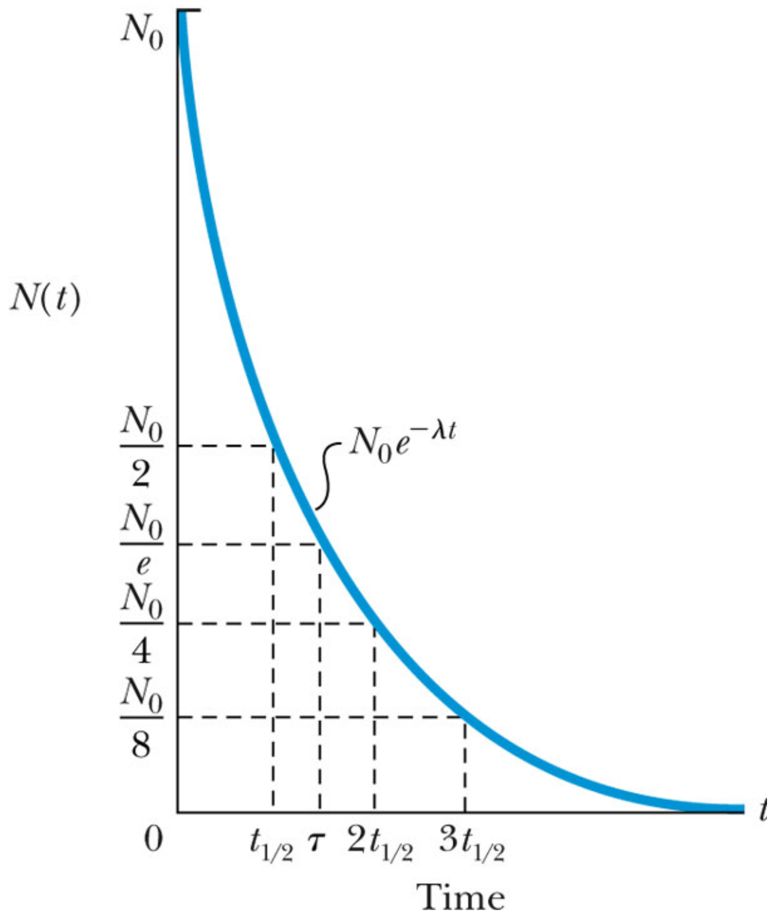
$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}$$

- The half-life is $t_{1/2} = \dfrac{-\ln(1/2)}{\lambda} = \dfrac{\ln(2)}{\lambda} = \dfrac{0.693}{\lambda}$

- The mean lifetime is $\tau = \dfrac{1}{\lambda} = \dfrac{t_{1/2}}{\ln(2)}$

6: Radioactive Decay

- The number of radioactive nuclei as a function of time



7: Alpha, Beta, and Gamma Decay

When a nucleus decays, all the conservation laws must be observed:

- Mass-energy
- Linear momentum
- Angular momentum
- Electric charge
- **Conservation of nucleons**
  - The total number of nucleons (*A*, the mass number) must be conserved in a low-energy nuclear reaction or decay.

7: Alpha, Beta, and Gamma Decay

- Let the radioactive nucleus ${}_{Z}^{A}X$ be called the parent and have the mass $M\left({}_{Z}^{A}X\right)$

- Two or more products can be produced in the decay.
- Let the lighter one be M_y and the mass of the heavier one (*daughter*) be M_D.
- The conservation of energy is $M\left({}_{Z}^{A}X\right) = M_D + M_y + Q/c^2$

  where Q is the energy released (**disintegration energy**) and equal to the total kinetic energy of the reaction products.

- If $B > 0$, a nuclide is bound and stable; $Q = \left[M\left({}_{Z}^{A}X\right) - M_D - M_y\right]c^2$
- If $Q > 0$, a nuclide is unbound, unstable, and may decay.
- If $Q < 0$, decay emitting nucleons do not occur.

Alpha Decay

- The nucleus ^{4}He has a binding energy of 28.3 MeV.
- If the last two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle (alpha decay) is possible.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}D + \alpha$$

$$Q = \left[M\left(^{A}_{Z}X \right) - M\left(^{A-4}_{Z-2}D \right) - M(^{4}\text{He}) \right] c^2$$

- If $Q > 0$, alpha decay is possible.

EX. $^{230}_{92}\text{U} \rightarrow \alpha + {}^{226}_{90}\text{Th}$

The appropriate masses are

$$M\left(^{230}_{92}\text{U} \right) = 230.033927 \text{ u}; \; M(^{4}\text{He}) = 4.002603 \text{ u}; \; M(^{226}_{90}\text{Th}) = 226.024891 \text{ u}$$

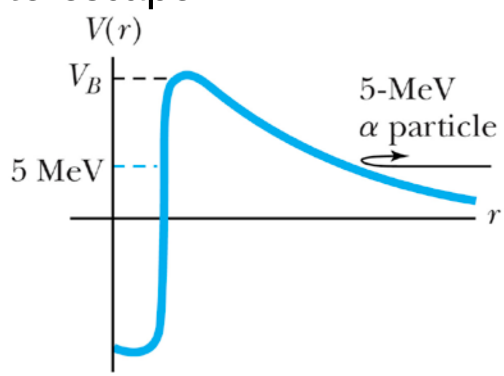
Alpha Decay

- Insert these masses into the following equation

$$Q = \left[M(^{230}\mathrm{U}) - M(^{226}\mathrm{Th}) - M(^{4}\mathrm{He})\right]c^2$$

$$= [230.033927\ \mathrm{u} - 226.024891\ \mathrm{u} - 4.002603\ \mathrm{u}]c^2\left(\frac{931.5\ \mathrm{MeV}}{c^2 \cdot \mathrm{u}}\right) = 6.0\ \mathrm{MeV}$$

- In order for alpha decay to occur, two neutrons and two protons group together within the nucleus prior to decay and the alpha particle has difficulty in overcoming the nuclear attraction from the remaining nucleons to escape.

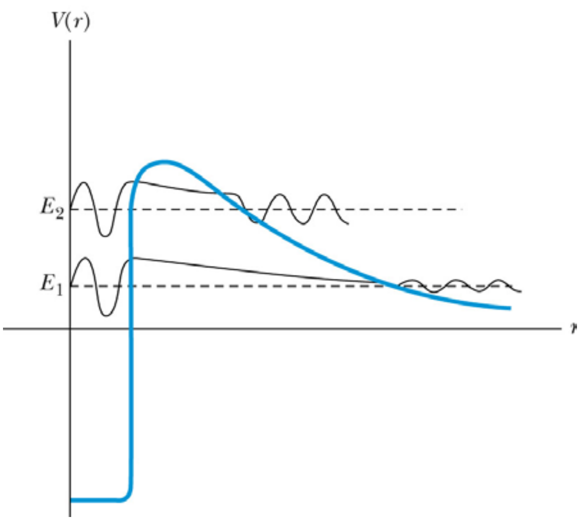


The potential energy diagram of alpha particle

Alpha Decay

- The barrier height V_B is greater than 20 MeV.
- The kinetic energies of alpha particles emitted from nuclei range from 4-10 MeV.

→ It is impossible classically for the alpha particle to reach the nucleus, but the alpha particles are able to tunnel through the barrier.

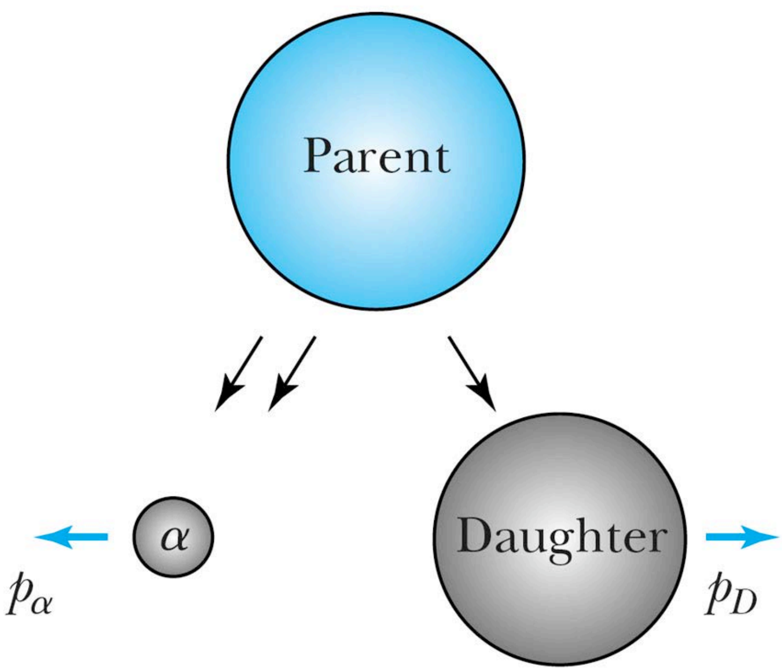


A higher energy E_2 has much higher probability than does a lower energy E_1.

There is a correlation between lower energies and greater difficulty of escaping (longer lifetimes).

Alpha Decay

- Assume the parent nucleus is initially at rest so that the total momentum is zero.
- The final momenta of the daughter p_D and alpha particle p_α have the same magnitude and opposite directions.



Alpha Decay

- From the conservation of energy and conservation of linear momentum, determine a unique energy for the alpha particle.

$$Q = K_\alpha + K_D$$

$$p_\alpha = p_D$$

$$K_\alpha = Q - K_D = Q - \frac{p_D{}^2}{2M_D} = Q - \frac{p_\alpha{}^2}{2M_D}$$

$$K_\alpha = Q - \frac{2M_a K_\alpha}{2M_D} = Q - \frac{M_\alpha}{M_D} K_\alpha$$

$$K_\alpha \left(1 + \frac{M_\alpha}{M_D}\right) = Q$$

$$K_\alpha = \frac{M_D}{M_D + M_\alpha} Q \approx \left(\frac{A-4}{A}\right) Q$$

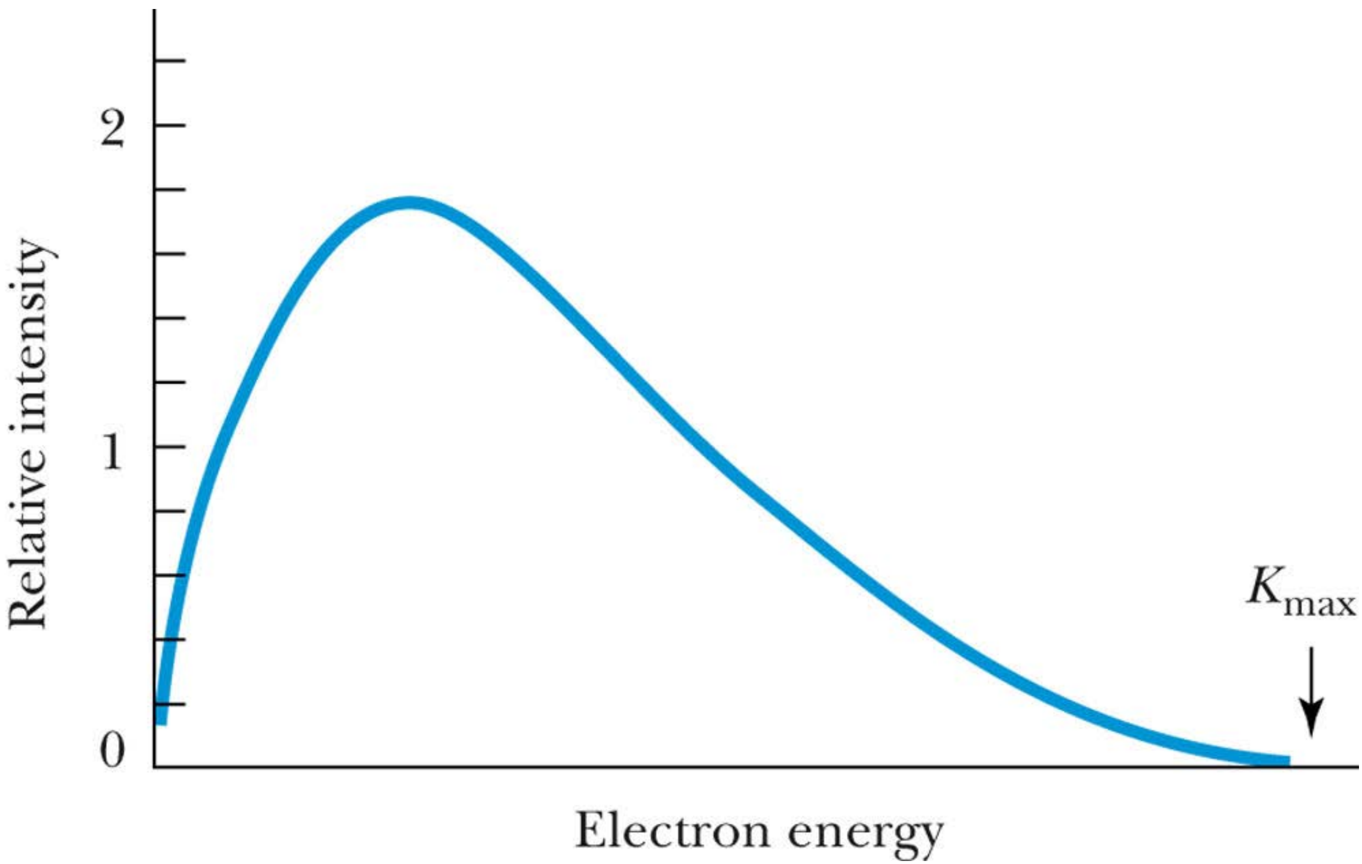
Beta Decay

- Unstable nuclei may move closer to the line of stability by undergoing beta decay.
- The decay of a free neutron is
- The beta decay of ^{14}C (unstable) to form ^{14}N, a stable nucleus, can be written as

$$n \rightarrow p + \beta^{-}$$

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^{-}$$

Beta Decay



The electron energy spectrum from the beta decay

Beta Decay

- There was a problem in neutron decay, the spin ½ neutron cannot decay to two spin ½ particles, a proton and an electron. ^{14}C has spin 0, ^{14}N has spin 1, and the electron has spin ½.

→ we cannot combine spin ½ & 1 to obtain a spin 0.

- Wolfgang Pauli suggested a **neutrino** ν that must be produced in beta decay. It has spin quantum number ½, charge 0, and carries away the additional energy required to satisfy the conservation of Energy.

Beta Decay

- An occasional electron is detected with the kinetic energy K_{max} required to conserve energy, but in most cases the electron's kinetic energy is less than K_{max}.

$\longrightarrow$ the neutrino has little or no mass, and its energy may be all kinetic.

- Neutrinos have no charge and do not interact *electromagnetically*.
- They are not affected by the *strong* force of the nucleus.
- They are the *weak* interaction.
- The electromagnetic and weak forces are the *electroweak* force.

β^- Decay

- There are *anti*neutrinos $\bar{\nu}$.
- The beta decay of a free neutron of ^{14}C is written as

$$n \rightarrow p + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$
$$^{14}\text{C} \rightarrow {}^{14}\text{N} + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$

- In the general beta decay of the parent nuclide $^A_Z X$ to the daughter $^{A}_{Z+1}D$, the reaction is $^A_Z X$

$$^A_Z X \rightarrow {}^{A}_{Z+1}D + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$

- The disintegration energy *Q* is

$$Q = \left[M\left(^A_Z X \right) - M\left(^{A}_{Z+1}D \right) \right] c^2 \qquad \beta^- \text{ decay}$$

- In order for β^- to occur, we must have $Q > 0$.
- The nucleus *A* is constant, but *Z* charges to $Z + 1$.

Beta Decay

- What happens for unstable nuclides with too many protons?
- Positive electron (positron) is produced.
- Positron is the antiparticle of the electron.
- A free proton does not decay when $t_{1/2} > 10^{32}$ y.
- The nucleus ^{14}O is unstable and decays by emitting a positron to become stable ^{14}N.

$$^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N} + \beta^{+} + \nu \qquad \beta^{+} \text{ decay}$$

- The general β^{+} decay is

$$^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;A}D + \beta^{+} + \nu \qquad \beta^{+} \text{ decay}$$

- The disintegration energy Q is

$$Q = \left[M\left({}^{A}_{Z}X \right) - M\left({}_{Z-1}^{\;\;A}D \right) - 2m_{e} \right] c^{2} \qquad \beta^{+} \text{ decay}$$

Electron Capture

- Classically, inner K-shell and L-shell electrons are tightly bound and their orbits are highly elliptical, these electrons spend a time passing through the nucleus, thereby the possibility of atomic electron capture.
- The reaction for a proton is $p + e^- \rightarrow n + \nu$
- The general reaction is

$$ {}^{A}_{Z}X + e^- \rightarrow {}_{Z-1}^{\;\;A}D + \nu \qquad \text{Electron capture} $$

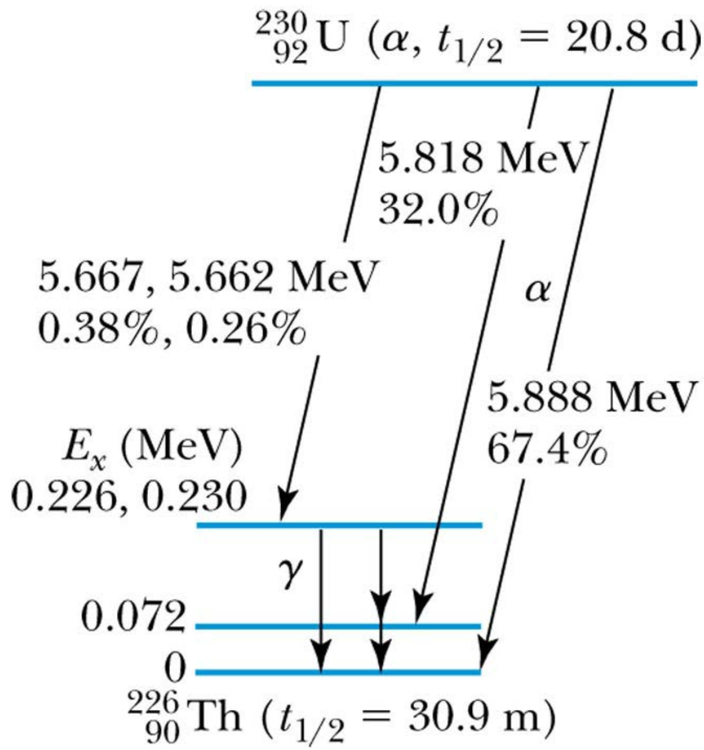
- The disintegration energy Q is

$$ Q = \left[M\left({}^{A}_{Z}X \right) - M\left({}_{Z-1}^{\;\;A}D \right) \right] c^2 \qquad \text{Electron capture} $$

Gamma Decay

- If the decay proceeds to an excited state of energy E_x rather than to the ground state, then Q for the transition to the excited state can be determined with respect to the transition to the ground state. The disintegration energy Q to the ground state Q_0.
- Q for a transition to the excited state E_x is

$$Q = Q_0 - E_x$$



Gamma Decay

- The excitation energies tend to be much larger, many keV or even MeV.
- The possibilities for the nucleus to rid itself of this extra energy is to emit a photon (gamma ray).
- The gamma-ray energy hf is given by the difference of the higher energy state $E_>$ and lower one $E_<$.

$$hf = E_> - E_<$$

- The decay of an excited state of $^{A}X^*$ (where * is an excited state) to its ground state is

$$^{A}X^* \rightarrow {}^{A}X + \gamma$$

- A transition between two nuclear excited states $E_>$ and $E_<$ is

$$^{A}X^*(E_>) \rightarrow {}^{A}X^*(E_<) + \gamma$$

Gamma Decay

- The gamma rays are normally emitted soon after the nucleus is created in an excited state.
- Sometimes selection rules prohibit a certain transition, and the excited state may live for a long time.
- These states are called **isomers** or **isomeric states** and are denoted by a small m for *metastable*.
- Ex: the spin 9 state of $^{210m}_{83}\mathrm{Bi}$ at 0.271 MeV excitation energy does not gamma decay because of a large spin difference transition.
- Even though $^{93m}_{41}\mathrm{Nb}$ is another example of prohibited (the probability of occurring is small) decay to the ground state, it does gamma decay.

8: Radioactive Nuclides

- The unstable nuclei found in nature exhibit natural radioactivity.

Table 12.2 Some Naturally Occurring Radioactive Nuclides

Nuclide	$t_{1/2}$ (y)	Natural Abundance
$^{40}_{19}K$	1.28×10^9	0.01%
$^{87}_{37}Rb$	4.8×10^{10}	27.8%
$^{113}_{48}Cd$	9×10^{15}	12.2%
$^{115}_{49}In$	4.4×10^{14}	95.7%
$^{128}_{52}Te$	7.7×10^{24}	31.7%
$^{130}_{52}Te$	2.7×10^{21}	33.8%
$^{138}_{57}La$	1.1×10^{11}	0.09%
$^{144}_{60}Nd$	2.3×10^{15}	23.8%
$^{147}_{62}Sm$	1.1×10^{11}	15.0%
$^{148}_{62}Sm$	7×10^{15}	11.3%

8: Radioactive Nuclides

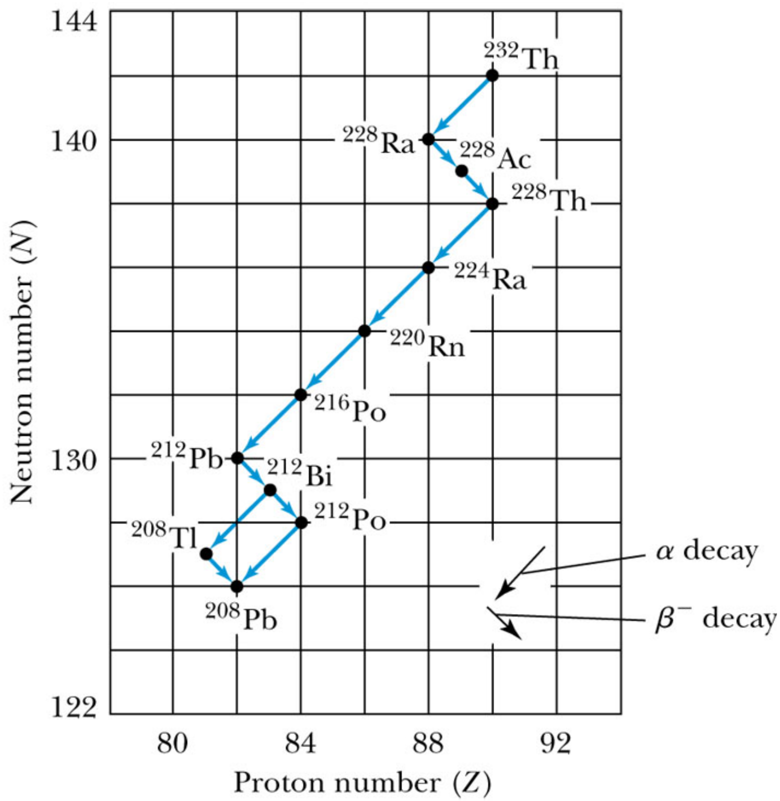
- The radioactive nuclides made in the laboratory exhibit artificial radioactivity.
- Heavy radioactive nuclides can change their mass number only by alpha decay (${}^{A}X \rightarrow {}^{A-4}D$) but can change their charge number Z by either alpha or beta decay.
- There are only four paths that the heavy naturally occurring radioactive nuclides may take as they decay.
- Mass numbers expressed by either:
  - $4n$
  - $4n + 1$
  - $4n + 2$
  - $4n + 3$

Table 12.3 The Four Radioactive Series

Mass Numbers	Series Name	Parent	$t_{1/2}$ (y)	End Product
$4n$	Thorium	${}^{232}_{90}Th$	1.40×10^{10}	${}^{208}_{82}Pb$
$4n + 1$	Neptunium	${}^{237}_{93}Np$	2.14×10^{6}	${}^{209}_{83}Bi$
$4n + 2$	Uranium	${}^{238}_{92}U$	4.47×10^{9}	${}^{206}_{82}Pb$
$4n + 3$	Actinium	${}^{235}_{92}U$	7.04×10^{8}	${}^{207}_{82}Pb$

8: Radioactive Nuclides

- The sequence of one of the radioactive series ^{232}Th



- ^{212}Bi can decay by either alpha or beta decay (*branching*).